Functional Stable Model Semantics, Answer Set Programming Modulo Theories, and Action Languages

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Abstract

"Answer Set Programming Modulo Theories (ASPMT)" is a recently proposed framework which tightly integrates answer set programming (ASP) and satisfiability modulo theories (SMT). Its mathematical foundation is the functional stable model semantics, an enhancement of the traditional stable model semantics to allow defaults involving functions as well as predicates. This talk will discuss how ASPMT can provide a way to overcome limitations of the propositional setting of ASP, how action language C+ can be reformulated in terms of ASPMT, and how it can be implemented based on the reformulation.

Introduction

Answer Set Programming (ASP)

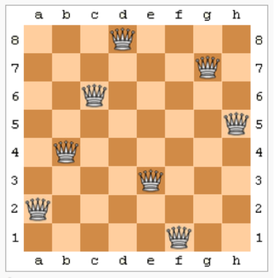
- Declarative programming paradigm. Suitable for knowledge intensive and combinatorial search problems.
- Theoretical basis: answer set semantics (Gelfond & Lifschitz, 1988).
- Expressive representation language: defaults, recursive definitions, aggregates, preferences, etc.
- ASP solvers:
  - SMODELS (Helsinki University of Technology, 1996)
  - DLV (Vienna University of Technology, 1997)
  - CMODELS (University of Texas at Austin, 2002)
  - PBMODELS (University of Kentucky, 2005)
  - CLASP (University of Potsdam, 2006) – winning first places at ASP'07/09/11/12, PB'09/11/12, and SAT'09/11/12
  - DLV-HEX computing HEX programs.
  - oClingo for reactive answer set programming.

Declarative Problem Solving using ASP

The basic idea is

- to represent the given problem by a set of rules,
- to find answer sets for the program using an ASP solver, and
- to extract the solutions from the answer sets.

N-Queens Puzzle in the Language of CLINGO




```
number(1..n).
#domain number(I).
#domain number(I1).
#domain number(J).
#domain number(J1).

% Each column has exactly one queen
1{q(K,J) : number(K)}1.

% Two queens cannot stay on the same row
:- q(I,J), q(I,J1), J<J1.

% Two queens cannot stay on the same diagonal
:- q(I,J), q(I1,J1), J<J1, #abs(I1-I)==J1-J.
```

Finding All Solutions for the 8-Queens Problem

With the command line

```
% clingo queens -c n=8 0
```

CLINGO computes and shows all 92 valid queen arrangements. For instance, the last one is

```
Answer: 92
q(5,8) q(7,7) q(2,6) q(6,5) q(3,4) q(1,3) q(8,2) q(4,1)
SATISFIABLE

Models       : 92
Time         : 0.010
  Prepare    : 0.000
  Prepro.    : 0.000
  Solving    : 0.010
```

Applications of ASP in AI

- planning ([Lif02], [DEF+03], [SPS09], [TSGM11], [GKS12])
- theory update/revision ([IS95], [FGP07], [OC07], [EW08], [ZCRO10], [Del10])
- preferences ([SW01], [Bre07], [BNT08a])
- diagnosis ([EFLP99], [BG03], [EBDT+09])
- learning ([Sak01], [Sak05], [SI09], [CSIR11])
- robotics ([CHO+09], [EHP+11], [AEEP11], [EHPU12], [APE12])
- description logics and semantic web ([EGRH06], [CEO09], [Sim09], [PHE10], [SW11], [EKSX12])
- data integration and question answering ([AFL10], [LGI+05])
- multi-agent systems ([VCP+05], [SPS09], [SS09], [BGSP10], [Sak11], [PSBG12])
- multi-context systems ([EBDT+09], [BEF11], [EFS11], [BEFW11], [DFS12])
- natural language processing/understanding ([BDS08], [BGG12], [LS12])
- argumentation ([EGW08], [WCG09], [EGW10], [Gag10])

Applications of ASP in Other Areas

- product configuration ([SN98], [TSNS03]) used by Variantum Oy
- Linux package configuration ([Syr00], [GKS11])
- wire routing ([ELW00], [ET01])
- combinatorial auctions ([BU01])
- game theory ([VV02], [VV04])
- decision support systems ([NBG+01]): used by United Space Alliance
- logic puzzles ([FMT02], [BD12])
- bioinformatics ([BCD+08], [EY09], [EEB10], [EEEO11])
- phylogenetics ([ELR06], [BEE+07], [Erd09], [EEEF09], [CEE11], [Erd11])
- haplotype inference ([EET09], [TE08])
- systems biology ([TB04], [GGI+10], [ST09], [TAL+10], [GSTV11])
- automatic music composition ([BBVF09],[BBVF11])
- assisted living ([MMB08], [MMB09], [MSMB11])
- team building ([RGA+12]): used by Gioia Tauro seaport
- software engineering ([EIO+11])
- bounded model checking ([HN03], [TT07])
- verification of cryptographic protocols ([DGH09])
- e-tourism ([RDG+10])

What Led to the Success of ASP?

- A simple, mathematically elegant semantics, based on the concept of a stable model
- Intelligent grounding—the process that replaces first-order variables with corresponding ground instances
- Efficient search methods that originated from propositional satisfiability solvers (SAT solvers).

Various Extensions

Starting from the Prolog syntax, the language of ASP has evolved:

- Strong negation [GLR91]
- Choice rules [SNS02]
- Aggregates [SNS02, FLP04, Fer05, PDB07, LM09, FL10], ...
- Preferences [BNT08b]
- Integration with CSP [Bal09, GOS09]
- Integration with SMT [JLN11]
- Integration with Description Logics [EIL+08, LP11]
- Probabilistic answer sets [BGR09]
- ...

Language Extension: Constraint Answer Set Programs (CASP)

Grounding is often the bottleneck. Solving is not applied until grounding is finished.

To alleviate the grounding bottleneck, integration of ASP with CSP/SMT solvers has been considered.

- Clingcon [GOS09]: Clasp + CSP solver Gecode

$$1 \leq amt(T) \leq 3 \leftarrow pour(T)$$
$$amt(T) = 0 \leftarrow not\ pour(T)$$
$$vol(T+1) = vol(T) + amt(T)$$

- EZCSP [Bal11]: Gringo + constraint solver SICStus Prolog or BProlog
- Dingo [JLN11]: Gringo + SMT solver Barcelogic

Fundamental Limitation Due to Lack of General Functions

ASP lacks general functions.

- Functional fluents in ASP are represented by predicates:

  $$WaterLevel(t+1, tank, l) \leftarrow WaterLevel(t, tank, l),\ not\ \ {\sim}WaterLevel(t+1, tank, l).$$

  Grounding generates a large number of instances as the domain gets larger.

- Using functions (e.g., $WaterLevel(t, tank) = l$) instead does not work because

  - Answer sets are Herbrand models: $WaterLevel(t+1, tank) = WaterLevel(t, tank)$ is always false.
  - Nonmonotonicity of ASP has to do with minimizing the predicates but has nothing to do with functions.

Fundamental Limitation Due to Lack of General Functions

- Even the constraint answer set sovers don't help. In CLINGCON this rule does not affect stable models.

$$\begin{aligned} &\mathit{WaterLevel}(t+1, \mathit{tank}) =^{\$} l \leftarrow \\ &\qquad \mathit{WaterLevel}(t, \mathit{tank}) =^{\$} l,\ \mathit{not}\ \mathit{WaterLevel}(t+1, \mathit{tank}) \neq^{\$} l. \end{aligned}$$

The lack of general functions in ASP is not only a disadvantage in comparison with other KR formalisms, but also a hurdle to cross over in integrating ASP with other declarative paradigms where functions are primitive constructs.

From Wikipedia Article on SMT

By comparison, answer set programming is also based on predicates *(more precisely, on atomic sentences created from atomic formula). Unlike SMT, answer-set programs do not have quantifiers, and cannot easily express constraints such as linear arithmetic or difference logic–ASP is at best suitable for boolean problems that reduce to the free theory of uninterpreted functions.*

ASP vs. SMT

- ASP is a successful nonmonotonic declarative programming paradigm, but is limited in handling first-order reasoning involving functions due to its propositional setting.
- SMT is a successful approach to solving some specialized first-order reasoning, but is limited in handling expressive nonmonotonic reasoning.

Answer Set Programming Modulo Theories (ASPMT)

- ASPMT tightly integrates ASP and SMT:

Monotonic	Nonmonotonic
FOL	Functional Stable Model Semantics [BL12]
SMT	ASP Modulo Theories [BL13]
SAT	Traditional ASP

- ASP = SAT + Loop formulas
- The syntax of ASPMT is the same as that of SMT. The semantics is defined as a special case of FSM [BL12].
- $$WaterLevel(t+1, tank) = l \leftarrow$$
  $$WaterLevel(t, tank) = l,\ \ not\ \ WaterLevel(t+1, tank) \neq l$$
  works under ASPMT.

My View

- The stable model semantics was successfully extended to the first-order level.
- Its computation can be carried out by compilation to declarative solvers, such as ASP solvers, CSP solvers, SMT solvers, ontology reasoners, or their combinations.
- Like SMT, we need to restrict to certain classes of first-order reasoning. ASP modulo theories will enjoy the expressiveness of the ASP modeling language while leveraging efficient constraint / theory solving methods available in SMT and other related computing paradigms.

This Talk is About

- First-order stable model semantics (FOSM).
- Extending FOSM to allow intensional functions
- ASPMT
- Reformulating $\mathcal{C}+$ in ASPMT

First-Order Stable Model Semantics

First-Order Stable Model Semantics [FLL07, FLL11]

Generalizes Gelfond and Lifschitz's 1988 definition of a stable model to first order sentences.

- Does not refer to grounding; not restricted to Herbrand models.
- Does not refer to reduct.
- Defined by a translation into second-order classical logic.

The stable models of F are defined as the models of F (in the sense of classical logic) that satisfy the "stability condition."

First-Order Stable Model Semantics

Idea 1: Treat logic programs as alternative notation for first-order formulas.

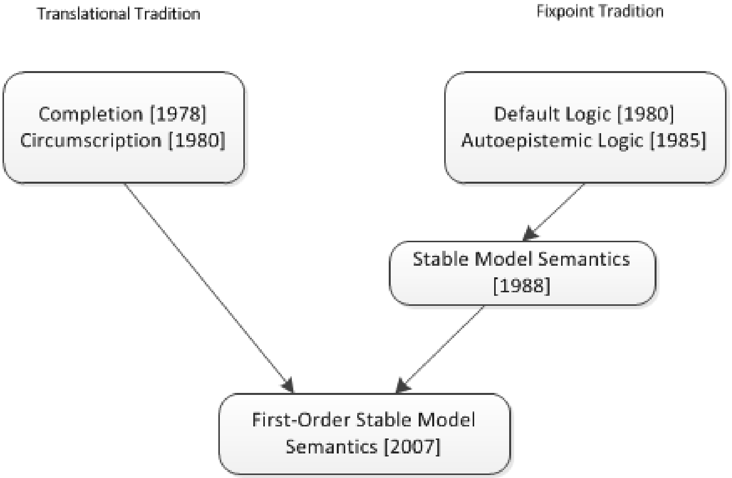
Logic program	FOL-representation
$p(X) \leftarrow \mathit{not}\ q(X), r(X)$	$\forall X(\neg q(X) \wedge r(X) \rightarrow p(X))$
$q(a)$	$\wedge q(a)$
$r(b)$	$\wedge r(b)$

Idea 2: Define the stable models of F as the models of

$$\mathrm{SM}[F; \mathbf{p}] = F \wedge (\text{2nd-order formula that enforces } \mathbf{p} \text{ to be } \textit{stable})$$

Similar to circumscription. (c.f. stability vs. minimality)

Translation vs. Fixpoint Traditions in Nonmonotonic Reasoning



Circumscription

The models of $\mathrm{CIRC}[F; \mathbf{p}]$ are the models of F that are minimal on $\mathbf{p}$. Formally,

$$\mathrm{CIRC}[F; \mathbf{p}] = F \wedge \neg \exists \mathbf{u}(\mathbf{u} < \mathbf{p} \wedge F(\mathbf{u}))$$

- $\mathbf{u}$: a list of distinct predicate variables similar to $\mathbf{p}$;
- $\mathbf{u} < \mathbf{p}$: a formula that expresses that $\mathbf{u}$ is strictly stronger than $\mathbf{p}$:
  - $u \leq p$ is defined as $\forall \mathbf{x}(u(\mathbf{x}) \rightarrow p(\mathbf{x}))$
  - $u = p$ is defined as $\forall \mathbf{x}(u(\mathbf{x}) \leftrightarrow p(\mathbf{x}))$
  - $\mathbf{u} < \mathbf{p}$ is defined as $(\mathbf{u} \leq \mathbf{p}) \wedge \neg(\mathbf{u} = \mathbf{p})$
- $F(\mathbf{u})$ is obtained from F by replacing all occurrences of $\mathbf{p}$ with $\mathbf{u}$.

First-Order Stable Model Semantics

The stable models of a first-order sentence F relative to a list $\mathbf{p}$ of (intensional) predicate constants are the models of

$$\mathrm{SM}[F;\mathbf{p}] = F \land \neg\exists\mathbf{u}(\mathbf{u} < \mathbf{p} \land F^*(\mathbf{u}))$$

$F^*(\mathbf{u})$ is defined as:

- $p_i(\mathbf{t})^* = u_i(\mathbf{t})$ if $p_i \in \mathbf{p}$
- for other atomic formula F, $\;F^* = F$
- $(\neg G)^* = \neg G^* \land \neg G$;
- $(G \odot H)^* = (G^* \odot H^*) \land (G \odot H) \qquad (\odot \in \{\land, \lor, \rightarrow\})$
- $(QxG)^* = QxG^* \land QxG \qquad (Q \in \{\forall, \exists\})$

If we drop the red parts, $F^*(\mathbf{u})$ becomes the same as $F(\mathbf{u})$, so SM becomes exactly the definition of CIRC.

First-Order Stable Model Semantics

The stable models of a first-order sentence F relative to a list $\mathbf{p}$ of (intensional) predicate constants are the models of

$$\mathrm{SM}[F; \mathbf{p}] = F \wedge \neg\exists \mathbf{u}(\mathbf{u} < \mathbf{p} \wedge F^*(\mathbf{u}))$$

$F^*(\mathbf{u})$ is defined as:

- $p_i(\mathbf{t})^* = u_i(\mathbf{t})$ if $p_i \in \mathbf{p}$
- for other atomic formula F, $\;F^* = F$
- $(\neg G)^* = \neg G^* \wedge \neg G$;
- $(G \odot H)^* = (G^* \odot H^*) \wedge (G \odot H) \qquad (\odot \in \{\wedge, \vee, \rightarrow\})$
- $(QxG)^* = QxG^* \wedge QxG \qquad (Q \in \{\forall, \exists\})$

If we drop the red parts, $F^*(\mathbf{u})$ becomes the same as $F(\mathbf{u})$, so SM becomes exactly the definition of CIRC.

Observe that $\mathbf{u} < \mathbf{p} \wedge F^*(\mathbf{u}) \rightarrow F$ is logically valid.

First-Order Stable Model Semantics

The stable models of a first-order sentence F relative to a list $\mathbf{p}$ of (intensional) predicate constants are the models of the second-order formula

$$\mathrm{SM}[F;\mathbf{p}] = F \wedge \neg\exists\mathbf{u}(\mathbf{u} < \mathbf{p} \wedge F^*(\mathbf{u}))$$

$F^*(\mathbf{u})$ can be simplified as:

- $p_i(\mathbf{t})^* = u_i(\mathbf{t})$ if $p_i \in \mathbf{p}$
- for other atomic formula F, $\;F^* = F$
- $(\neg G)^* = \neg G$
- $(G \odot H)^* = (G^* \odot H^*) \qquad (\odot \in \{\wedge, \vee\})$
- $(G \to H)^* = (G^* \to H^*) \wedge (G \to H)$
- $(QxG)^* = QxG^* \qquad (Q \in \{\forall, \exists\})$

$\wedge$, $\vee$, $\forall$, $\exists$ are monotone

$\quad$(e.g., $\mathbf{u} < \mathbf{p} \wedge (F^*(\mathbf{u}) \wedge G^*(\mathbf{u})) \to (F \wedge G)$)

$\neg$ is anti-monotone

$\quad$(e.g., $\mathbf{u} < \mathbf{p} \wedge \neg F \to \neg F^*(\mathbf{u})$)

$\to$ is neither monotone nor anti-monotone

Example

$$\mathrm{SM}[F; \mathbf{p}] = F \wedge \neg\exists \mathbf{u}(\mathbf{u} < \mathbf{p} \wedge F^*(\mathbf{u}))$$

$$F = p(a) \wedge \forall x(p(x) \wedge \neg r(x) \rightarrow q(x))$$

$$F^*(u, v, w) = \quad u(a) \wedge \forall x(u(x) \wedge \neg r(x) \rightarrow v(x)) \wedge (p(x) \wedge \neg r(x) \rightarrow q(x))$$

$\mathrm{SM}[F; p, q, r]$ is equivalent to

$$F \wedge \neg\exists uvw((u, v, w) < (p, q, r) \wedge u(a) \wedge \forall x(u(x) \wedge \neg r(x) \rightarrow v(x))$$

which can be written in FOL

$$\forall x(p(x) \leftrightarrow x = a) \wedge \forall x(q(x) \leftrightarrow p(x) \wedge \neg r(x)) \wedge \forall x(r(x) \leftrightarrow \bot)$$

c.f. $\mathrm{CIRC}[F; p, q, r]$ is equivalent to

$$F \wedge \neg\exists uvw((u, v, w) < (p, q, r) \wedge u(a) \wedge \forall x(u(x) \wedge \neg w(x) \rightarrow v(x))$$

Relating SM to Traditional ASP

Theorem

The stable models of a logic program Π according to the 1988 definition are precisely the Herbrand models of $\mathrm{SM}[\Pi; pr(\Pi)]$.

Example

$\{p(a), q(a)\}$ is the unique

- stable model of $\left\{\begin{array}{l} p(a) \\ q(x) \leftarrow p(x), not\ r(x) \end{array}\right.$ under the 1988 definition
- Herbrand model of $\mathrm{SM}[p(a) \wedge \forall x(p(x) \wedge \neg r(x) \rightarrow q(x)); p, q, r]$.

System F2LP [LP09] for Computing Herbrand Stable Models of First-Order Formulas

The input languages of ASP solvers do not allow complex formulas.

F2LP is a front-end to ASP solvers that turns first-order formulas into logic program syntax.

- `f2lp [input-program] | clingo`

The F2LP rule

```
t(X) <- v(X) & not ?[Y]:e(X,Y)
```

describes the set t of terminal vertices (the symbol `?` represents the existential quantifier).

Stable Models and Circumscription

Theorem

The stable model semantics and circumscription coincide on the class of canonical formulas [LP12b].

In other words, minimal models and stable models coincide on canonical formulas.

The theorem allows us to reformulate the Event Calculus, the Situation Calculus, and Temporal Action Logics in ASP, and use ASP solvers to compute them

- Event Calculus in ASP [KLP09, LP12b]
- Situation Calculus in ASP [LP10, LP12b]
- Integrating rules and ontologies in FOSM [LP11]
- Temporal Action Logics in ASP [LP12a]

Event Calculus in Terms of ASP

Theorem

$$
\begin{array}{ll}
& \mathrm{CIRC}[\Sigma; \mathit{Initiates}, \mathit{Terminates}, \mathit{Releases}] \wedge \mathrm{CIRC}[\Delta; \mathit{Happens}] \wedge F \\
\iff & \quad \{\textit{By the theorem on canonical formulas}\} \\
& \mathrm{SM}[\Sigma; \mathit{Initiates}, \mathit{Terminates}, \mathit{Releases}] \wedge \mathrm{SM}[\Delta; \mathit{Happens}] \wedge F \\
\iff & \quad \{\textit{By the splitting theorem}\} \\
& \mathrm{SM}[\Sigma \wedge \Delta \wedge F; \mathit{Initiates}, \mathit{Terminates}, \mathit{Releases}, \mathit{Happens}]
\end{array}
$$

F2LP turns $\Sigma \wedge \Delta \wedge F$ into the input language of ASP solvers.

http://reasoning.eas.asu.edu/ecasp

http://decreasoner.sourceforge.net/csr/ecas/

ASP-based vs. SAT-based Approach

- DEC reasoner is based on the reduction of circumscription to completion. Able to solve 11 out of 14 benchmark problems.
- ECASP can handle the *full* version of the event calculus (modulo grounding). Able to solve all 14 problems.
- For example, the following axiom cannot be handled by the DEC reasoner, but can be done by the ASP approach.

$$
\begin{aligned}
&\mathit{HoldsAt}(\mathit{HasBananas}, t)\\
&\wedge \mathit{Initiates}(e, \mathit{At}(\mathit{Monkey}, l), t) \rightarrow \mathit{Initiates}(e, \mathit{At}(\mathit{Bananas}, l), t)
\end{aligned}
$$

- ECASP computes faster.

Experiments

Problem (max. time)	DEC reasoner	ECASP w/ LPARSE + CMODELS	ECASP w/ GRINGO + CLASP	ECASP W CLINGO
Falling w/ AntiTraj (15)	270.2 (269.3+0.9) A:416/C:3056	0.74 (0.66+0.08) A:5757/R:10480/C:0	0.10 (0.08+0.02) A:4121/R:7820	0.08
Falling w/ Events (25)	107.70 (107.50+0.20) A:1092/C:12351	34.77 (30.99+3.78) A:1197/R:390319/C:1393	2.90 (2.01+0.89) A:139995/R:208282	2.32
HotAir Balloon (15)	61.10 (61.10+0.00) A:288/C:1163	0.19 (0.16+0.03) A:489/R:2958/C:678	0.04 (0.03+0.01) A:1137/R:1909	0.03
Telephone1 (40)	18.00 (17.50+0.50) A:5419/C:41750	1.70 (1.51+0.19) A:23978/R:30005/C:0	0.31 (0.26+0.05) A:21333/R:27201	0.25

A: number of atoms, C: number of clauses, R: number of ground rules

DEC reasoner and CMODELS used the same SAT solver RELSAT.

$$
\begin{array}{ll}
 & \mathrm{CIRC}[\Gamma_{occ}; \mathit{Occurs}] \wedge \mathrm{CIRC}[\Gamma_{depc} \wedge \Gamma_{acs}; \mathit{Occlude}] \;\; \wedge \Gamma_{rest} \\
\Longleftrightarrow & \quad \{\text{By the theorem on canonical formulas}\} \\
 & \mathrm{SM}[\Gamma_{occ}; \mathit{Occurs}] \wedge \mathrm{SM}[\Gamma_{depc} \wedge \Gamma_{acs}; \mathit{Occlude}] \;\; \wedge \Gamma_{rest} \\
\Longleftrightarrow & \quad \{\text{By the symmetric splitting theorem}\} \\
 & \mathrm{SM}[\Gamma_{occ} \wedge \Gamma_{depc} \wedge \Gamma_{acs} \wedge \Gamma_{rest}; \mathit{Occurs}, \mathit{Occlude}] \; .
\end{array}
$$

System F2LP turns the formula $\Gamma_{occ} \wedge \Gamma_{depc} \wedge \Gamma_{acs} \wedge \Gamma_{rest}$ into the input language of ASP solvers.

Integrating TAL with Ontologies

Integrating TAL with description logics is straightforward as they are both based on classical logic (**c.f.** integrating rules and ontologies).

Example

A patient is suffering from gastritis and complains of abdominal pain. Is aspirin recommended?

- In OWL ontology (e.g., National Drug File):
  - Aspirin may treat pain
  - Gastrointestinal bleeding is a contraindication for administering aspirin.
- In TAL: Gastritis causes gastrointestinal bleeding.

Answer: No, since gastrointestinal bleeding is a contraindication.

Representing the Example in TAL (Part)

Gastritis usually causes gastrointestinal bleeding:

$$\textbf{dep1}\ \forall t([t]\ Cond(Gastritis) \wedge \neg Ab_2(Gastritis) \rightarrow R([t+1]\ Cond(GastrointestinalBleeding))).$$

Domain rules for drug recommendation:

$$\textbf{acs1}\ [t_1, t_2]\ Administer(d) \rightarrow \Big([t_1]\ Cond(c) \wedge \neg Ab_1(d, c) \wedge (\mathtt{MAY\_TREAT}(d, c) \vee \mathtt{MAY\_PREVENT}(d, c)) \wedge \neg\exists c_1, t\big(t_1 \leq t \leq t_2 \wedge [t]\ Cond(c_1) \wedge \mathtt{CONTRAINDICATION}(d, c_1)\big) \rightarrow I((t_1, t_2]\ \neg Cond(c))\Big).$$

$$\textbf{acs2}\ [t_1, t_2]\ Administer(d) \rightarrow \forall t\big(t_1 \leq t \leq t_2 \wedge [t]\exists c(Cond(c) \wedge \mathtt{CONTRAINDICATION}(d, c)) \rightarrow I([t, t_2]\ SideEffect(d))\big).$$

$\mathtt{MAY\_TREAT}(Aspirin, Pain)$ and $\mathtt{CONTRAINDICATION}(Aspirin, GastrointestinalBleeding)$ can be obtained from NDF ontology by using the DLVHEX rdf plugin.

Reductive Semantics [LLP08]

- A simple, alternative approach to understanding the meaning of counting and choice in answer set programming by reducing them to first order formulas.

$$\{q(x)\} \leftarrow p(x) \quad \Rightarrow \quad \forall x(p(x) \rightarrow (q(x) \vee \neg q(x)))$$

$$r \leftarrow \#count\{x : p(x)\} \geq 2$$

$$\Rightarrow \quad (\exists xy(p(x) \wedge p(y) \wedge \neg(x = y))) \rightarrow r$$

Generalized Quantifiers

The approach doesn't cover other extensions of the stable model semantics

- Arbitrary aggregates
- (Abstract) constraints
- External atoms
- Description logic atoms

A common issue is how to incorporate "complex atoms."

$\forall$ and $\exists$ are just two instances of a much more general concept of quantifiers [Mos57]. A generalized quantifier (GQ) can represent any relation over relations.

We extended FOSM to allow generalized quantifiers and showed that all the extensions above can be viewed in terms of generalized quantifiers under FOSM [LM12a, LM12b, LM12c].

Stable Models of Formulas with Intensional Functions

Leaking Container Example

Describe a water tank that has a leak but that can be refilled to the maximum amount, say 10, with the action *FillUp*.

Functional Stable Model Semantics (FSM) [Bartholomew and Lee, 2012]

Allows for assigning default values to non-Herbrand functions, which is useful for expressing inertia and default behaviors of systems.

Leaking Container Example

$$\begin{array}{rcl} \{Amount_1 = x\} & \leftarrow & Amount_0 = x+1 \\ Amount_1 = 10 & \leftarrow & FillUp\ . \end{array}$$

$\{F\}$ is a choice rule standing for $F \vee \neg F$

- $I_1 = \{FillUp = \mathrm{F}, Amount_0 = 6, Amount_1 = 5\}$:
  I_1 is a stable model of F (relative to $Amount_1$) as well as a model.
- $I_2 = \{FillUp = \mathrm{F}, Amount_0 = 6, Amount_1 = 8\}$:
  I_2 is a model of F but not a stable model.
- $I_3 = \{FillUp = \mathrm{T}, Amount_0 = 6, Amount_1 = 10\}$:
  I_3 is a model of F as well as a stable model of F.

Infinitary Ground Formula w.r.t. an Interpretation

Since the universe may be infinite, grounding a first-order sentence F relative to an interpretation I (denoted $gr_I[F]$) may introduce infinite conjunctions and disjunctions.

Leaking Container Example. $gr_I[F]$ is

$$
\begin{array}{rcl}
\{Amount_1 = 0\} & \leftarrow & Amount_0 = 0+1 \\
\{Amount_1 = 1\} & \leftarrow & Amount_0 = 1+1 \\
 & \dots & \\
Amount_1 = 10 & \leftarrow & FillUp
\end{array}
$$

Reduct-based Definition of FSM

For any two interpretations I, J of the same signature and any list $\mathbf{c}$ of distinct predicate and function constants, we write $J <^{\mathbf{c}} I$ if

- J and I have the same universe and agree on all constants not in $\mathbf{c}$;
- $p^J \subseteq p^I$ for all predicate constants p in $\mathbf{c}$; and
- J and I do not agree on $\mathbf{c}$.

The reduct $F^{\underline{I}}$ of an infinitary ground formula F relative to an interpretation I is the formula obtained from F by replacing every maximal subformula that is not satisfied by I with $\bot$.

I is a stable model of F relative to $\mathbf{c}$ if

- I satisfies F, and
- every interpretation J such that $J <^{\mathbf{c}} I$ does not satisfy $(gr_I[F])^{\underline{I}}$.

Leaking Container Example

$I_1 = \{\mathit{FillUp}=\mathrm{F}, \mathit{Amount}_0=6, \mathit{Amount}_1=5\} \models \mathrm{SM}[F; \mathit{Amount}_1]$

$$
\begin{array}{lrcl}
gr_{I_1}(F): & \mathit{Amount}_1=0 \lor \neg(\mathit{Amount}_1=0) & \leftarrow & \mathit{Amount}_0=0+1 \\
 & & \cdots & \\
 & \mathit{Amount}_1=5 \lor \neg(\mathit{Amount}_1=5) & \leftarrow & \mathit{Amount}_0=5+1 \\
 & & \cdots & \\
 & \mathit{Amount}_1=10 & \leftarrow & \mathit{FillUp} \\
\hline
(gr_{I_1}[F])^{\underline{I_1}}: & \bot \lor \neg\bot & \leftarrow & \bot \\
 & & \cdots & \\
 & \mathit{Amount}_1=5 \lor \bot & \leftarrow & \mathit{Amount}_0=5+1 \\
 & & \cdots & \\
 & \bot & \leftarrow & \bot
\end{array}
$$

No J such that $J <^{\mathit{Amount}_1} I_1$ satisfies the reduct.

Leaking Container Example

$I_2 = \{FillUp{=}\mathrm{F}, Amount_0{=}6, Amount_1{=}8\} \not\models \mathrm{SM}[F; Amount_1]$

$$
\begin{array}{lrcl}
gr_{I_2}(F): & Amount_1{=}0 \vee \neg(Amount_1{=}0) & \leftarrow & Amount_0{=}0{+}1 \\
 & & \cdots & \\
 & Amount_1{=}5 \vee \neg(Amount_1{=}5) & \leftarrow & Amount_0{=}5{+}1 \\
 & & \cdots & \\
 & Amount_1{=}10 & \leftarrow & FillUp \\
\hline
(gr_{I_2}[F])^{\underline{I_2}}: & \bot \vee \neg\bot & \leftarrow & \bot \\
 & & \cdots & \\
 & \bot \vee \neg\bot & \leftarrow & Amount_0{=}5{+}1 \\
 & & \cdots & \\
 & \bot & \leftarrow & \bot
\end{array}
$$

I_2 satisfies the reduct, but there are also other interpretations J such that $J <^{Amount_1} I_2$ that satisfy the reduct.

Leaking Container Example

$I_3 = \{FillUp = \mathrm{T}, Amount_0 = 6, Amount_1 = 10\} \models \mathrm{SM}[F; Amount_1]$

$$
\begin{array}{llcl}
gr_{I_3}(F): & Amount_1 = 0 \vee \neg(Amount_1 = 0) & \leftarrow & Amount_0 = 0+1 \\
& & \cdots & \\
& Amount_1 = 5 \vee \neg(Amount_1 = 5) & \leftarrow & Amount_0 = 5+1 \\
& & \cdots & \\
& Amount_1 = 10 & \leftarrow & FillUp \\
\hline
(gr_{I_3}[F])^{\underline{I_3}}: & \bot \vee \neg\bot & \leftarrow & \bot \\
& & \cdots & \\
& \bot \vee \neg\bot & \leftarrow & Amount_0 = 5+1 \\
& & \cdots & \\
& Amount_1 = 10 & \leftarrow & FillUp
\end{array}
$$

No J such that $J <^{Amount_1} I_3$ satisfies the reduct.

FSM in Terms of SOL

$\mathbf{c}$ is a list of predicate and function constants called intensional.

$\mathbf{u}$ is a list of predicate and function variables corresponding to $\mathbf{c}$.

$\mathrm{SM}[F;\ \mathbf{c}]$ is defined as

$$F \wedge \neg\exists\mathbf{u}(\mathbf{u} < \mathbf{c} \wedge F^*(\mathbf{u}))$$

- For predicate symbols (variables or constants) u and c
  - $u \leq c$ is defined as $\forall\mathbf{x}(u(\mathbf{x}) \rightarrow c(\mathbf{x}))$
  - $u = c$ is defined as $\forall\mathbf{x}(u(\mathbf{x}) \leftrightarrow c(\mathbf{x}))$

- For function symbols u and c,
  - $u = c$ is defined as $\forall\mathbf{x}(u(\mathbf{x}) = c(\mathbf{x}))$

- $\mathbf{u} < \mathbf{c}$ is defined as $(\mathbf{u}^{pred} \leq \mathbf{c}^{pred}) \wedge \neg(\mathbf{u} = \mathbf{c})$

FSM in Terms of SOL

The stable models of a first-order sentence F relative to a list of distinct predicate and function constants $\mathbf{c}$ are the models of the second-order formula

$$\mathrm{SM}[F;\ \mathbf{c}] = F \wedge \neg\exists\mathbf{u}(\mathbf{u} < \mathbf{c} \wedge F^*(\mathbf{u}))$$

where $F^*(\mathbf{u})$ is defined as:

- when F is an atomic formula, F^* is $F(\mathbf{u}) \wedge F$;
- $(G \wedge H)^* = G^* \wedge H^*$; $\quad (G \vee H)^* = G^* \vee H^*$;
  $(G \rightarrow H)^* = (G^* \rightarrow H^*) \wedge (G \rightarrow H)$;
- $(\forall x G)^* = \forall x G^*$; $\quad (\exists x F)^* = \exists x F^*$.

Blocks World in FSM

$$
\begin{array}{rcl}
\bot & \leftarrow & Loc(b_1, t) = b \wedge Loc(b_2, t) = b \wedge (b_1 \neq b_2) \\
Loc(b, t+1) = l & \leftarrow & Move(b, l, t) \\
\bot & \leftarrow & Move(b, l, t) \wedge Loc(b_1, t) = b \\
\bot & \leftarrow & Move(b, b_1, t) \wedge Move(b_1, l, t) \\
\{Loc(b, 0) = l\} & & \\
\{Move(b, l, t)\} & & \\
\{Loc(b, t+1) = l\} & \leftarrow & Loc(b, t) = l \ .
\end{array}
$$

The last rule is a default formula that describes the commonsense law of inertia.

Blocks World : Eliminating Function *Loc*

For the class of **c**-plain formulas, intensional functions can be eliminated in favor of intensional predicates.

$$
\begin{array}{rcl}
\bot & \leftarrow & Loc(b_1, b, t) \land Loc(b_2, b, t) \land \neg(b_1 = b_2) \\
Loc(b, l, t+1) & \leftarrow & Move(b, l, t) \\
\bot & \leftarrow & Move(b, l, t) \land Loc(b_1, b, t) \\
\bot & \leftarrow & Move(b, b_1, t) \land Move(b_1, l, t) \\
\{Loc(b, l, 0)\} & & \\
\{Move(b, l, t)\} & & \\
\{Loc(b, l, t+1)\} & \leftarrow & Loc(b, l, t) \\
 & & \\
\bot & \leftarrow & Loc(b, l, t) \land Loc(b, l_1, t) \land \neg(l = l_1) \\
\bot & \leftarrow & \neg\exists l\ Loc(b, l, t)
\end{array}
$$

Blocks World: In the Language of ASP Solvers

```
% every block is a location
location(B) :- block(B).

% the table is a location
location(table).

:- 2{loc(BB,B,ST): block(BB)}.

loc(B,L,T+1) :- move(B,L,T).

% preconditions
:- move(B,L,T), loc(B1,B,T).
:- move(B,B1,T), move(B1,L,T).
```

```
{loc(B,L,0)}.

{move(B,L,T)}.

{loc(B,L,T+1)} :- loc(B,L,T).

% uniqueness constraint
:- 2{loc(B,LL,ST): location(LL)}.

% existence constraint
:- {loc(B,LL,ST): location(LL)}0.
```

Answer Set Programming Modulo Theories

Answer Set Programming Modulo Theories (ASPMT)

- Defined as FSM with the fixed interpretation for the background signature.

  Let σ^{bg} be the (many-sorted) signature of a background theory bg, and let J^{bg} be the (fixed) interpretation of σ^{bg}. Let σ be a signature that is disjoint from the background signature σ^{bg}.

  An interpretation I of σ is a model of an SMT sentence F w.r.t. the background theory bg, denoted by $I \models_{bg} F$, if $I \cup J^{bg}$ satisfies F.

  I is a stable model of F relative to $\mathbf{c}$ (w.r.t. background theory σ^{bg}) if $I \models_{bg} \mathrm{SM}[F; \mathbf{c}]$.

Completion : Turning ASPMT to SMT

Theorem

For any sentence F in Clark normal form that is tight on **c**, *an interpretation I that satisfies $\exists xy(x \neq y)$ is a stable model F iff I is a model of the completion of F.*

Leaking Container Example, Continued.

$$\begin{array}{rcl} \{Amount_1 = x\} & \leftarrow & Amount_0 = x+1 \\ Amount_1 = 10 & \leftarrow & FillUp\ . \end{array}$$

can be rewritten as

$$Amount_1 = x \quad \leftarrow \quad (\neg\neg(Amount_1 = x) \wedge Amount_0 = x+1) \vee \ (x = 10 \wedge FillUp)$$

and completion turns it into

$$Amount_1 = x \quad \leftrightarrow \quad (\neg\neg(Amount_1 = x) \wedge Amount_0 = x+1) \vee \ (x = 10 \wedge FillUp).$$

The formula can be written without mentioning the variable x:

$$\big((Amount_0 = Amount_1 + 1) \vee (Amount_1 = 10 \wedge FillUp)\big) \wedge (FillUp \rightarrow Amount_1 = 10)$$

In the language of SMT solver iSAT, this formula can be represented as

```
(Amt' + 1 = Amt) or (Amt' = 10 and FillUp);
FillUp -> Amt' = 10;
```

In the language of SMT solver Z3, this formula can be represented as

```
(assert (or (= (+ Amt1 1) Amt0) (and (= Amt1 10) FillUp)))
(assert (=> FillUp0 (= Amt1 10)))
```

Gears World Example

$$
\begin{array}{rcl}
M1Speed_t = x & \leftarrow & M1Speed_{t-1} = x - 1 \wedge IncreaseM1_{t-1} \\
M1Speed_t = x & \leftarrow & M1Speed_{t-1} = x \wedge \neg\neg(M1Speed_t = x) \\
 & \ldots & \\
Gear1Speed_t = \; x & \leftarrow & Higher_t = x \wedge Connected_t .
\end{array}
$$

Instance Size	ASP (Gringo+Clasp) Execution		iSAT Execution	
	Run Time (Grounding + Solving)	Atoms	Run Time	Variable
5	.02s (.02s + 0s)	3174	.03s	331
10	.3s (.3s + 0s)	10161	.19s	596
20	9.46s (4.02s + 5.11s)	36695	.79s	1126
30	42.56s (22.32s + 20.24s)	77627	2.05s	1656
50	923.74s (297.26 + 626.48s)	207706	14.35s	2716
100	out of memory		494.77s	5366

Comparison with Constraint Answer Set Solving

- CLINGCON programs [GOS09] can be viewed as a special case of ASPMT instances, which allows non-Herbrand functions, but does not allow them to be intensional.
- ASP(LC) programs by [LJN12] can be viewed similarly.
- In fact, they can be viewed even as a special case of the language from [FLL11], which FSM properly generalizes.

Reformulating $\mathcal{C}+$ in ASPMT

Language $\mathcal{C}+$ [GLL+04]

- $\mathcal{C}+$ is a formal model of parts of natural language for representing and reasoning about transition systems.
- Can represent actions with conditional and indirect effects, nondeterministic actions, and concurrently executed actions.
- Can represent multi-valued fluents, defined fluents, additive fluents, and rigid constants.
- Can represent defeasible causal laws and action attributes.
- Implemented in systems CCALC, CPLUS2ASP, COALA.

Reformulating $\mathcal{C}+$ in ASPMT

- Language $\mathcal{C}+$ is an expressive action description language but its semantics was defined in terms of propositional causal theories, which limits the language to express discrete changes only.
- By reformulating $\mathcal{C}+$ in terms of ASPMT, we can apply $\mathcal{C}+$ for reasoning about continuous changes as well, and use SMT solvers to compute the language.

causal laws	In ASPMT
caused F **if** G	$\neg\neg\ i:G \rightarrow i:F$
caused F **if** G **after** H	$\neg\neg\ (i+1):G \wedge i:H \rightarrow (i+1):F$

Representing Continuous Changes in Enhanced $\mathcal{C}+$

We distinguish between steps and real clock times. We assume the Theory of Reals as the background theory, and introduce

- *Time*: a simple fluent constant with value sort $\mathcal{R}_{\geq 0}$ (clock time);
- *Dur*: an action constant with value sort $\mathcal{R}_{\geq 0}$, which denotes the time elapsed between the two consecutive states.

We postulate:

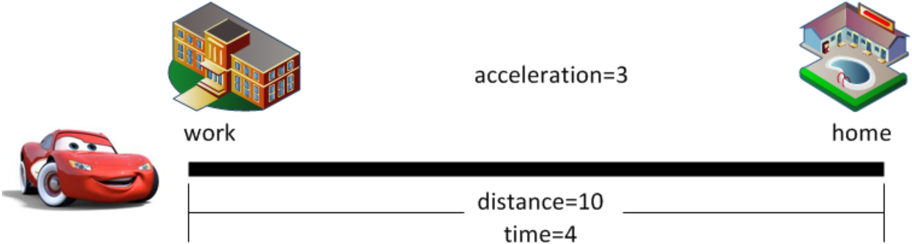
$$\begin{array}{l} \textbf{caused } \mathit{Time}=t \textbf{ if } \mathit{Time}=t \\ \textbf{caused } \mathit{Dur}=t \textbf{ if } \mathit{Dur}=t \\ \textbf{caused } \bot \textbf{ if } \neg(\mathit{Time}=t+t') \textbf{ after } \mathit{Time}=t \wedge \mathit{Dur}=t' \end{array}$$

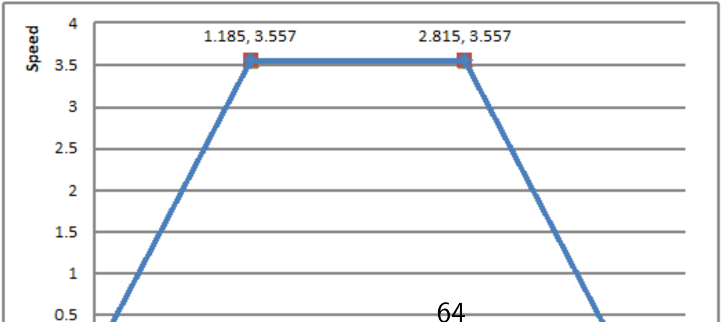
Continuous changes can be described as a function of duration using fluent dynamic laws

$$\textbf{caused } c=f(\mathbf{x},\mathbf{x}',t) \textbf{ if } \mathbf{c}'=\mathbf{x}' \textbf{ after } (\mathbf{c}=\mathbf{x}) \wedge (\mathit{Dur}=t) \wedge G$$

Planning with Continuous Time

Example: give a formal representation of the domain to generate a plan





Car Example in Enhanced $\mathcal{C}+$

Notation: d, v, v', t, t' are variables of sort $\mathcal{R}_{\geq 0}$; A, MS are real numbers.

Simple fluent constants:	Domains:
Speed, *Distance*, *Time*	$\mathcal{R}_{\geq 0}$
Action constants:	Domains:
Accelerate, *Decelerate*	Boolean
Dur	$\mathcal{R}_{\geq 0}$

Causal laws:

caused $\mathit{Speed} = v + \mathrm{A} \times t$ **after** $\mathit{Accelerate} \wedge \mathit{Speed} = v \wedge \mathit{Dur} = t$
caused $\mathit{Speed} = v - \mathrm{A} \times t$ **after** $\mathit{Decelerate} \wedge \mathit{Speed} = v \wedge \mathit{Dur} = t$
caused $\mathit{Distance} = d + 0.5 \times (v + v') \times t$ **if** $\mathit{Speed} = v'$
after $\mathit{Distance} = d \wedge \mathit{Speed} = v \wedge \mathit{Dur} = t$
constraint $\mathit{Time} = t + t'$ **after** $\mathit{Time} = t \wedge \mathit{Dur} = t'$
constraint $\mathit{Speed} \leq \mathrm{MS}$

inertial *Speed*
exogenous *Time*
exogenous c for every action constant c

Turning $\mathcal{C}+$ into ASPMT and SMT

$$\mathcal{C}+ \xrightarrow{\textit{semantics}} \mathrm{ASPMT} \xrightarrow{\textit{completion}} \mathrm{SMT} \xrightarrow{\textit{eliminating variables}} \mathrm{SMT\ solvers}$$

In $\mathcal{C}+$:
caused $\mathit{Speed} = v + \mathrm{A} \times t$ **after** $\mathit{Accelerate} \wedge \mathit{Speed} = v \wedge \mathit{Dur} = t$
caused $\mathit{Speed} = v - \mathrm{A} \times t$ **after** $\mathit{Decelerate} \wedge \mathit{Speed} = v \wedge \mathit{Dur} = t$
caused $\mathit{Speed} = v$ **if** $\mathit{Speed} = v$ **after** $\mathit{Speed} = v$

In ASPMT:
$i{+}1 : \mathit{Speed} = x \ \leftarrow\ (x = v + \mathrm{A} \times t) \wedge\ i : (\mathit{Accelerate} \wedge \mathit{Speed} = v \wedge \mathit{Dur} = t)$
$i{+}1 : \mathit{Speed} = x \ \leftarrow\ (x = v - \mathrm{A} \times t) \wedge\ i : (\mathit{Decelerate} \wedge \mathit{Speed} = v \wedge \mathit{Dur} = t)$
$i{+}1 : \mathit{Speed} = x \leftarrow \neg\neg(i{+}1 : \mathit{Speed} = x) \wedge i : \mathit{Speed} = x$

In SMT: The completion on $i{+}1 : \mathit{Speed}$ yields a formula that is equivalent to

$$\begin{aligned} i{+}1 : \mathit{Speed} = x \leftrightarrow\ & \big(x = (i : \mathit{Speed} + A \times i : \mathit{Dur}) \ \wedge\ i : \mathit{Accelerate}\big) \\ & \vee\ \big(x = (i : \mathit{Speed} - A \times i : \mathit{Dur}) \ \wedge\ i : \mathit{Decelerate}\big) \\ & \vee\ \big(i{+}1 : \mathit{Speed} = x \ \wedge\ i : \mathit{Speed} = x\big). \end{aligned}$$

In the language of SMT Solvers

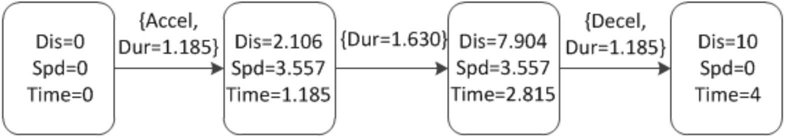
Variable x in the formula can be eliminated by equivalent transformations using equality:

$$i\!:\!Accelerate \rightarrow i\!+\!1\!:\!Speed = (i\!:\!Speed + A \times i\!:\!Dur)$$

$$i\!:\!Decelerate \rightarrow i\!+\!1\!:\!Speed = (i\!:\!Speed - A \times i\!:\!Dur)$$

$$\begin{aligned} &(i\!+\!1 : Speed = (i\!:\!Speed + A \times i\!:\!Dur) \wedge i\!:\!Accelerate) \\ \vee\ &(i\!+\!1 : Speed = (i\!:\!Speed - A \times i\!:\!Dur) \wedge i\!:\!Decelerate) \\ \vee\ &(i\!:\!Speed = i\!+\!1\!:\!Speed)\ . \end{aligned}$$

The shortest step plan found by SMT solver iSAT:
(http://isat.gforge.avacs.org)



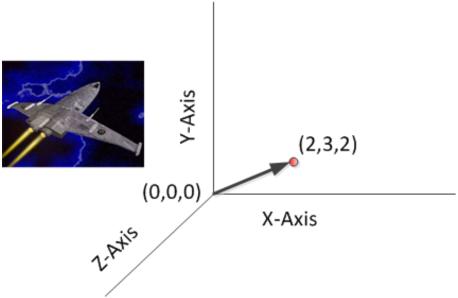
Reasoning about Indirect Effects

Indirect effects can be represented in static causal laws in $\mathcal{C}+$:

- For example, Accelerating and decelerating not only affect the speed and the distance of the car, but also indirectly affect the speed and the distance of the bag in the car.

$$\begin{array}{l}\textbf{caused } \mathit{Speed}(\mathit{Bag}) = x \textbf{ if } \mathit{Speed} = x \wedge \mathit{In}(\mathit{Bag}, \mathit{Car}) \\ \textbf{caused } \mathit{Distance}(\mathit{Bag}) = x \textbf{ if } \mathit{Distance} = x \wedge \mathit{In}(\mathit{Bag}, \mathit{Car})\ .\end{array}$$

Reasoning about Additive Fluents



Describe the cumulative effects of firing multiple jets:

- In the language of CCALC:
  $\mathit{Fire}(j)$ **increments** $\mathit{Vel}(ax)$ **by** n/Mass **if** $\mathit{Force}(j, ax) = n$
  limited to integer arithmetic.
- In enhanced $\mathcal{C}+$:
  $\mathit{Fire}(j)$ **increments** $\mathit{Vel}(ax)$ **by** $n/\mathit{Mass} \times t$ **if** $\mathit{Force}(j, ax) = n \wedge \mathit{Dur} = t$.

Even for integer domains, computing $\mathcal{C}+$ using SMT solvers was more effective than CCALC (which uses SAT solvers) and CPLUS2ASP (which uses ASP solvers).

Max Step	CCalc		CPLUS2ASP		$\mathcal{C}$+ in iSAT v1.0	
	Run Time (grounding+solving)	# of atoms / clauses	Run Time (grounding+solving)	# of atoms / rules	Run Time last/total	# of variables / clauses (bool + real)
1	0.16 (0.12+0.00)	488 / 1872	0.005 (0.005+0)	1864 / 2626	0/0	(42+53) / 182
2	0.57 (0.40+0.00)	3262 / 14238	0.033 (0.033+0)	6673 / 12035	0/0	(82+98) / 352
3	10.2 (2.62+6)	32772 / 155058	0.434 (0.234+0.2)	42778 / 92124	0/0	(122+143) / 520
4	505.86 (12.94+479)	204230 / 992838	12.546 (3.176+9.37)	228575/ 503141	0/0	(162+188) / 688
5	failed (51.10+failed)	897016 / 4410186	73.066 (15.846+57.22)	949240/ 2060834	0/0.03	(202+233) / 856
6	time out	–	3020.851 (62.381+2958.47)	3179869/ 6790167	0/0.03	(242+278) / 1024
10	time out	–	time out	–	0.03/0.09	(402+458) / 1696
50	time out	–	time out	–	0.09/1.39	(2002+2258) / 8416
100	time out	–	time out	–	0.17/5.21	(4002+4508) / 16816
200	time out	–	time out	–	0.33/21.96	(8002+9008) / 33616

The enhanced $\mathcal{C}+$ is flexible enough to represent the start-process-end model, where instantaneous actions may initiate or terminate processes.

Example: Two Taps Water Tank with Leak
TurnOn(x) **causes** *On*$(x) \wedge$ *Dur* $= 0$
TurnOff(x) **causes** *On*$(x) = \mathrm{F} \wedge$ *Dur* $= 0$

On(x) **increments** *Level* **by** $\mathrm{W}(x) \times t$ **if** *Dur* $= t$
Leaking **increments** *Level* **by** $-(\mathrm{V} \times t)$ **if** *Dur* $= t$

constraint $(\mathrm{Low} \leq \mathit{Level}) \wedge (\mathit{Level} \leq \mathrm{High})$
inertial *On*(x), *Leaking*
exogenous c for every action constant c

exogenous *Time*
constraint $\mathit{Time} = t + t'$ **after** $\mathit{Time} = t \wedge \mathit{Dur} = t'$

Conclusion

- ASPMT is a natural formalism that combines the advantages of ASP and SMT. Enhancements in ASP and SMT can be carried over to ASPMT.
- We expect that many results known between ASP and SAT can be carried over to the relationship between ASPMT and SMT. Completion is one such example.
- The enhanced C+, defined by a reduction to ASPMT, allows us to handle reasoning about hybrid systems, where discrete state changes and continuous changes coexist.
- See Related Presentations at IJCAI.
  - Functional Stable Model Semantics and Answer Set Programming Modulo Theories.
  - Answer Set Programming Modulo Theories and Reasoning about Continuous Changes.
  - Action Language BC: Preliminary Report.

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